Geometry optimization of PMSMs comparing full and fractional pitch winding configurations for aerospace actuation applications

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Optimization of electromechanical aerospace actuators requires a multi-objective and comparative analysis in order to account for performance and manufacturing cost terms. This paper introduces a particular optimization methodology presenting stable convergence characteristics which has been applied to optimize the geometry of both Fractional Slot Concentrated Winding (FSCW) and Full Pitch Concentrated Winding (FPCW) permanent magnet motor configurations. The proposed algorithm combines technical and physical advantages of the FSCWs and FPCWs into an optimally shaped stator-winding configuration. The resultant motor design has been validated through a prototype and experimental results illustrated its suitability for aerospace actuation applications.

Index Terms—Actuators, Aerospace engineering, Design optimization, Finite element methods, Permanent magnet motors.

I. INTRODUCTION

CURRENT trends for more electric aircraft technology favor Fractional Slot Concentrated Winding (FSCW) Permanent Magnet Synchronous Machine (PMSM) actuators [1]. FSCW actuators are intended to gradually replace hydraulic flight controls, allowing for both safer and "more-green" operations [2]. This is mainly due to the advantages of low cogging torque, short end turns, high slot fill factor, as well as fault tolerance and flux-weakening favoring FSCWs to Full Pitch Concentrated Windings (FPCWs) in the respective applications [3]. However, fractional pitch winding configuration affects the performance of the actuator similarly to the case of a reduced number of turns in the winding, leading to back-EMF voltage amplitude and power-factor reduction when compared to the case of FPCWs [4]. Such a performance related effect can be critical in aerospace applications requiring actuation systems with the ability to comply with maximum efficiency and high power density specifications. As FSCW technical characteristics call for less copper, while FPCW physical characteristics call for maximum EMF per inductor, the optimum actuator topology depends on the application specifications and has to be determined analytically [5], [6].

This paper introduces a particular optimization algorithm, facilitating the comparative approach on the stator geometry optimization of surface PMSMs involving FPCW and FSCW configurations. More specifically, a Rosenbrock based optimization algorithm is introduced in order to minimize an application-specific penalty function through a Sequential Unconstrained Minimization Technique (SUMT) [7], [8]. The proposed formula of the penalty function includes efficiency-performance related terms, as well as technical terms, related to the manufacturing cost of each stator configuration. Proper sigma terms (penalty terms) ensuring no violation of the optimization constraints have also been introduced.

The optimization algorithm offered stable convergence characteristics in all FPCW and FSCW design cases considered. The overall performance improvement of the optimized design has been validated through measurements on a prototype.

II. ACTUATOR MODELING

In a first step, an estimation of the actuator structure is achieved by considering classical machine design techniques. Although such an analytical approach does not enable detailed design optimization, due to the approximative electromagnetic field representation, it delivers a sub-optimum set of design variables, within a region of the global optimum. In a further step, such an approach enables the use of fast and robust local optimizers, adopted to handle complex optimization.

Table I summarizes the basic properties of the surface mounted PMSM prototype, which are common in all the FPCW and FSCW configurations compared.

<table>
<thead>
<tr>
<th>MACHINE PROTOTYPE DESIGN CHARACTERISTICS (DIMENSIONS IN MM)</th>
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<tbody>
<tr>
<td><strong>General</strong></td>
</tr>
<tr>
<td>Number of phases 3</td>
</tr>
<tr>
<td>Number of poles 20</td>
</tr>
<tr>
<td>Motor active length 100</td>
</tr>
<tr>
<td><strong>Rotor</strong></td>
</tr>
<tr>
<td>Magnet inner radius 32.75</td>
</tr>
<tr>
<td>Magnet (rotor) outer radius 35.75</td>
</tr>
<tr>
<td>Magnet angle 19.125 deg</td>
</tr>
<tr>
<td><strong>Gap</strong></td>
</tr>
<tr>
<td>Gap width 0.50</td>
</tr>
<tr>
<td><strong>Stator</strong></td>
</tr>
<tr>
<td>Stator outer radius 50</td>
</tr>
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</table>

Finally, parametric 2D FEMs are introduced. An FPCW topology and two FSCW topologies involving non-overlapping both alternative teeth wound and all teeth wound configurations have been modeled.

A. FPCW topology

The FPCW topology produces the largest possible EMF for a given number of inductors in the winding. One pole of the actuator has been modeled by using appropriate anti-periodic lateral boundary conditions (3334 nodes). Fig. 1a shows the triangular mesh employed and the magnetic flux distribution for a sub-optimum actuator design at nominal load. Fig. 1b shows the respective magnetic vector potential values, illustrating the magnetic loading of one pole of the FPCW actuator.
Fig. 1. 2D FEM of one pole of the FPCW PMSM. (a) mesh employed and magnetic flux distribution (b) computed magnetic vector potential.

B. FSCW, non-overlapping topology

This configuration can be mainly combined with two types of stator winding, which span the airgap as described in Fig. 2.

Fig. 2. FSCW stator winding configurations. (a) non-overlapping, alternate teeth wound (b) non-overlapping, all teeth wound.

Figs. 3a and 3b show the respective FEMs of the alternative teeth wound (FSCW1) and all teeth wound (FSCW2) configurations (34353 and 26588 nodes respectively). Fig. 4 shows the airgap magnetic vector potential variation with the mechanical angle, depicting its spatial non-uniformity.

Fig. 3. 2D FEM and flux distribution of the FSCW PMSM. (a) FSCW1 (b) FSCW2.

Fig. 4. Airgap magnetic vector potential values in FSCW configurations.

The number of stator slots in each case has been determined analytically, in order to enable optimal electromagnetic coupling of the stator and 20-pole rotor MMF vectors (maximum mean torque).

III. PROPOSED OPTIMIZATION ALGORITHM

Recent research on optimization procedures focuses on the application of search algorithms, both stochastic and deterministic. Stochastic search algorithms, such as genetic and differential evolution algorithms, mainly operate as global optimizers. However, they require many iterations, as they address the starting point problem by multisampling the cost function randomly [9]. On the other hand, deterministic search algorithms necessitate less computational effort and exhibit fast and robust convergence at least to one local optimum even under multi-objective constrained optimization [10].

The proposed methodology focuses on the multi-objective and comparative optimization of FSCW versus FPCW actuators. Initially a sub-optimum actuator design is defined by employing analytical and FEM based design techniques. In a second step a particular aggregate yet detailed optimization technique enables further tuning of the design variables. Such a two step procedure facilitates convergence of geometry optimization in aerospace applications involving strict and multiple technical specifications.

Based on the above framework, a Rosenbrock based optimization algorithm is introduced as illustrated in Fig. 5. [11].

Fig. 5. Flowchart of the proposed optimization algorithm.

The Rosenbrock method is a 0th order direct-search algorithm and it does not require gradient of the target function. However, this algorithm approximates a gradient search by combining advantages of 0th order and 1st order strategies. Moreover, constraints handling through the SUMT increases stability in regard to the penalty function methods [12].

Fig. 6. Optimization procedure block diagram.
though convenient Matlab scripts. The penalty function at the beginning of \(k^{th}\)-iteration is given in eq. (1).

\[
P^i(X_k) = G_1 \frac{T_{\text{mean}}(X_k)}{T_{\text{mean}}(X_0)} + G_2 \frac{T_{\text{ripple}}(X_k)}{T_{\text{ripple}}(X_0)} + G_3 A_{\text{norm}}(X_k) + \\
+ G_4 \frac{T_{\text{mean}}(X_k)}{T_{\text{mean}}(X_0)} \frac{T_{\text{mean}}(X_0)}{T_{\text{mean}}(X_0)} + G_5 C(X_k) \frac{C(X_k)}{C(X_0)} + R^i \sum_{i=1}^{I} l_i g_i(X_k), \quad R^i > 0
\]

where: \( X_k = [L_{\text{tooth}}, W_{t1}, W_{t2}] \) (see also Fig 6)

\( G_i - G_5 \) weight coefficients \( \in (0,1) \)

\( T_{\text{mean}}, T_{\text{ripple}} \): mean, ripple torque (N*m)

\( P_{\text{cop}}, \) copper loss (W)

\( A_{\text{norm}} \): 3rd and 5th h. EMF normalized amplitudes sum

\( g_i(X_k): \) constraints of the form \( g_i(X_k) \geq 0, i=1,2,3 \)

and,

\[
C(X_k) = C \left( \frac{T_{\text{mean}}}{L} \frac{W_{t1}}{W_{t2}}, \frac{W_{t1}}{W_{t2}} \right) + C_2 \left( \frac{W_{t1}}{W_{t2}} \right)
\]

(2)

An important novelty of the proposed penalty function formulation relies on the constitution of the particular cost terms \( C_1, C_2 \) composing the total technical cost, \( C \), related to the stator winding manufacturing process. These terms are associated with the winding fill factor and the slot shape respectively. The variation of these cost terms with the respective design variables is shown in Figs. 7a and 7b.

**IV. RESULTS - A COMPARATIVE APPROACH**

The proposed methodology has been applied for the optimization of FPCW, FSCW1 and FSCW2 actuators. Weights \( G_1,G_5 \) have been set in order to give equal priorities to technical as well as physical optimization of the actuator.

The applied algorithm offered stable and fast convergence in all cases considered. Fig. 8 illustrates the variation of the normalized penalty function values during optimization runs for the three cases considered. Moreover, technical costs variation is shown in Fig. 9.

**Fig. 7. Technical cost terms variation. (a) \( C_1 \) terms. Fill factor effect (b) \( C_2 \) term. Stator tooth-slot shape effect.**

**Fig. 8. Iterative display of penalty function values.**

**Fig. 9. Technical cost (C) terms variation during optimization runs.**

**Fig. 10. Mean torque and torque ripple as a function of tooth width.**

Fine-tuned values of the optimization variables are shown in Table II. Table III summarizes characteristic output parameter values of the geometrically optimized actuators.

**TABLE II**

<table>
<thead>
<tr>
<th>Var</th>
<th>FPCW</th>
<th>FSCW1</th>
<th>FSCW2</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( L_{\text{tooth}} ) (mm)</td>
<td>( W_{t1} ) (A)</td>
<td>( P_{\text{cop.}} ) (W)</td>
</tr>
<tr>
<td></td>
<td>9.25 (100.54)</td>
<td>9.33 (103.67)</td>
<td>9.26 (102.89)</td>
</tr>
<tr>
<td></td>
<td>2.50 (113.6)</td>
<td>5.6 (97.5)</td>
<td>4.2 (105)</td>
</tr>
<tr>
<td></td>
<td>7.34 (110)</td>
<td>7.67 (98.33)</td>
<td>6.83 (105.08)</td>
</tr>
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</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Top'gy</th>
<th>( T_{\text{mean}, \text{ripple}} ) (Nm)</th>
<th>EMF (%)</th>
<th>( I_{\text{rms}} ) (A)</th>
<th>( P_{\text{cop.}} ) (W)</th>
<th>( T_{\text{mean}} ) (Nm)</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPCW</td>
<td>31.5 / 0.70</td>
<td>3.2</td>
<td>4.4</td>
<td>20.0</td>
<td>600</td>
<td>1.28</td>
</tr>
<tr>
<td>FSCW1</td>
<td>31.8 / 1.10</td>
<td>8.8</td>
<td>3.9</td>
<td>18.0</td>
<td>485</td>
<td>1.44</td>
</tr>
<tr>
<td>FSCW2</td>
<td>34.0 / 0.65</td>
<td>5.6</td>
<td>0.8</td>
<td>16.5</td>
<td>475</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Compared to FPCW and FSCW1 geometrically optimized configurations, the FSCW2 topology offers the maximum torque per copper loss square root ratio, combined with the minimum ripple and back-EMF harmonic content. Besides, surface permanent magnet architecture has been specified, which forces a stiff relation between system efficiency and copper loss. Consequently, superior actuation performance and efficiency is expected in the FSCW2 actuator case.

However, FSCW2 presents a 52% greater optimum technical cost with respect to the FSCW1 topology. This is mainly due to the use of double layer windings, which involves increased fill factor. It may be noted that generally technical costs terms are not prevailing in this type of applications [1], [2].

**V. EXPERIMENTAL TESTING AND VALIDATION**

In a next step, a 3D electromagnetic and thermal model of the FSCW2 actuator has been developed. Fig. 11a shows the tetrahedral 3D mesh employed (88831 nodes). The computed steady-state temperature distribution corresponding to full load conditions is shown in Fig. 11b.
This figure illustrates the influence of the stator design on the physical-thermal characteristics of the actuator. As the winding losses constitute the main heat source, stator multi-objective design optimization is of crucial importance.

The optimized FSCW2 topology performance has been validated by measurements on a prototype shown in Fig. 12.

The measured terminal voltage as well as the measured torque time variations with phase current are shown in Fig. 13. Fig. 14 summarizes temperature readings, in the case of a varying load profile, depicting the actuator thermal performance.

The comparison of simulated and experimental results, concerning torque and EMF harmonics for different loading conditions, is tabulated in table IV.

The non-negligible discrepancy between the simulated and measured 5th harmonic EMF observed, can be attributed to the dimensional tolerance errors of permanent magnets integrated in the prototype, which lead to limited rotor eccentricity [13]. However, the overall performance characteristics of the simulation and experimental results are in good agreement, confirming the design methodology suitability for the specific class of applications.

VI. CONCLUSION

A particular methodology for the comparative optimization of FPCW and FSCW PMSM actuators has been introduced. The latter has been applied to optimize the most favorable candidate PMSM configurations for aerospace actuation applications and enabled improved assessment of combined technical-performance related costs. The resulting actuator architecture achieves suitable performance-efficiency characteristics for this class of applications, as illustrated both through simulation investigations and experimental validation.

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